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AN ANALYSIS OF THREE POST
LAUNCH EVASION STRATEGIES

by

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June 1984

Approved for public release, distribution unlimited.
Prepared for: Strategic Systems Projects Office,
Arlington, VA 20376

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER NPS-71-84-001	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) An Analysis of Three Post Launch Evasion Strategies		5. TYPE OF REPORT & PERIOD COVERED Technical
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) R. N. Forrest		8. CONTRACT OR GRANT NUMBER(s) N0003083AF31319
9. PERFORMING ORGANIZATION NAME AND ADDRESS Naval Postgraduate School Monterey, CA 93943		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS Strategic Systems Projects Office 1931 Jefferson Davis Highway Arlington, VA 20376		12. REPORT DATE June 1984
		13. NUMBER OF PAGES 21
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release, distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Submarine Evasion Strategies
Stratetic Submarine Vulnerability
Target Speed Distributions
Target Velocity Distribution

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The report addresses three post launch evasion strategies. A velocity distribution which results in a uniform position between two concentric circles at any time is derived in an appendix.

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I. Introduction

This report contains an analysis of three post launch evasion strategies. The purpose of the analysis is to provide equations that can be used with values determined by other means to obtain a conservative estimate of the relative effectiveness of the three strategies under specified conditions. The measure of effectiveness that is used in the analysis is the probability that, because of the localization resulting from a launch, a launching submarine will be damaged by an opponent's weapon. The effectiveness of an evasion strategy depends on the number of weapons that will be used by an opponent against an evading submarine and on the time and the location at their detonation. The effectiveness also depends on the weapon damage range and the maximum evasion speed. Factors which constrain the number of weapons that could be used by an opponent and the maximum evasion speed of a submarine and factors which determine weapon damage range are not considered in the analysis. The number of weapons, the weapon damage range and the maximum evasion speed are parameter values that must be determined from other sources in order to use the equations that determine the value of the measure of effectiveness for the three strategies.

II. A Post Launch Threat Model

The post launch threat model that is used for the analysis in this report is defined by the following assumptions: Detection of an acoustic, visual or infrared transient signal associated with a ballistic missile launch and the precise localization of the launch point by an opponent is certain. A launch signal jeopardizes a launching submarine to the degree that the submarine is localized by its detection. In particular, after a launch, at a time unknown to the launching submarine, an opponent will precisely place and simultaneously detonate n weapons in the launch area and each weapon will have a damage radius equal to r_c. Launch area geography will not restrict a launching submarine's choice of an evasive motion strategy to diminish this threat. However, the opponent knows a submarine's maximum evasion speed u_M, the submarine's evasive motion strategy, and the delay time t between missile launch and weapon detonation. Consequently, at weapon detonation, the launching submarine's poisition will be known by the opponent to be on a disc whose boundary is a localization circle of radius $u_M t$ that is centered on the launch site.

The analysis in the following sections is applicable to a cruise missile launch as well as to a ballistic missile launch. However, for cruise missile launch one might want to consider uncertain detection of the launch signals. One might also need to consider the possibility of remote launch from an ejected capsule launch container. In this case, the more stringent condition that the submarine's location at the time of launch was known would be required.

III. The First Evasive Motion Strategy

The first evasion motion strategy is based on the criterion: At the time of weapon placement make your bearing from the launch site as uncertain as possible and make your range from the launch site as large as possible. It is defined as follows: After missile launch, move away from the launch site at an optimum depth and at an evasion speed equal to u_M on a course chosen so that each course between 000° and 360° is equally likely. The first weapon placement strategy is based on the criterion: Make p as large as possible given a launching submarine uses the first motion strategy. It is defined as follows: At an optimum depth, place n weapons at points on a circle that is centered on the launch site and is of radius $r = [(u_M t)^2 - r_c{}^2]^{\frac{1}{2}}$. Choose the points so that weapon damage circles do not overlap. This implies that weapons are placed at the centers of nonintersecting chords of the launching submarine's localization circle of length $2r_c$. Consequently, each weapon damage circle encloses an arc of length $s = 2(u_M t) \sin^{-1} (r_c/u_M t)$ on the launching submarine's localization circle; and this is the maximum arc length that can be enclosed by a weapon damage circle. The geometry for a single weapon is shown in Figure 1. Since the location of the launching submarine is equally likely to be at any point on the localization circle, the probability that the launching submarine will be on or within the weapon damage circle is

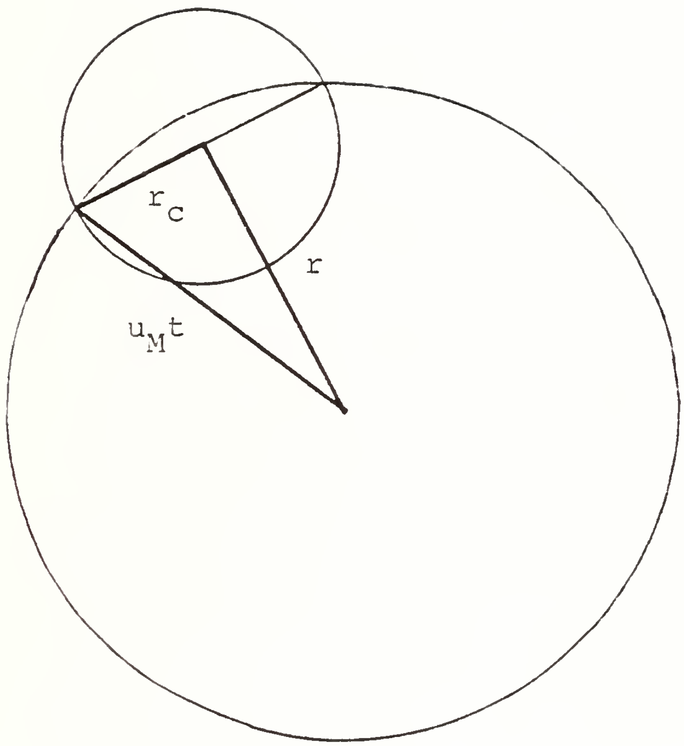

Figure 1. The first weapon placement strategy geometry for a single weapon.

determined by the following equation:

$$(1) \qquad p = (n/\pi) \sin^{-1} (r_C/u_M t)$$

given the weapon placement strategy can be implemented. A necessary and sufficient condition for this is that $n \sin^{-1}(r_C/u_M t) \leq \pi$. By analysis in Reference 1, it is shown that in the sense of game theory, for n = 1 and $u_M t/\sqrt{2} \leq r_C < u_M t$, the first motion strategy and the first weapon placement strategy are the solutions to the game established by the threat model. The analysis also implies that the strategies are the solutions for $n \leq \pi/\sin^{-1}(r_C/u_M t)$.

IV. The Second Evasive Motion Strategy

The second evasive motion strategy is based on the criterion: Make your position at the time of the weapon placement as uncertain as possible. For the second evasive motion strategy, a launching submarine's position will be uniformly distributed on a disc of radius $u_M t$ that is centered on the launch site. It differs from the first evasive motion strategy in that, in addition to its course, the submarine's speed is also a random quantity. Its distribution, which is triangular, is described in Appendix 1. The second weapon placement strategy is based on the criterion: Make p as large as possible given a launching submarine uses the second motion strategy. It is defined as follows: Place n weapons at points on or within a circle of radius $u_M t - r_c$ that is centered on the launch site subject to the constraint that weapon damage circles cannot overlap. For the second evasive motion strategy, at the time of weapon detonation a launching submarine is located at a random point which is uniformly distributed on a localization disc whose boundary is the localization circle of radius $u_M t$ that is centered on the launch site. Each weapon determines a weapon damage disc of radius r_c that is included within the localization disc. Since weapon damage circles do not overlap, the area of the localization disc covered by the weapon damage discs is $n\pi r_c^2$. Consequently, the probability p that the launching submarine will be on or within a weapon damage circle is $n r_c^2/(u_M t)^2$. Since the maximum area of the localization disc that can be covered by n weapons is $n\pi r_c^2$, p is a maximum and is determined by the following equation:

$$(2) \qquad p = n r_c^2/(u_M t)^2$$

given the weapon placement strategy can be implemented. A necessary condition for this is the $n r_c^2 < (u_M t)^2$ or equivalently $r_c < u_M t/\pi\sqrt{n}$. Consequently, for $n \geq 2$, the first evasive motion strategy and the first weapon placement strategies are not optimal strategies. However, Danskin's analysis in Reference 2 in effect implies that given a submarine's speed is to remain constant during the evasive motion, the second evasive motion strategy and the second weapon placement strategy are optimal in a practical sense. This is the case even though the analysis corresponds to a sequential detonation of weapons rather than a simultaneous one.

V. The Third Evasive Motion Strategy

Suppose that an opponent's weapon placement strategy is assumed to include the following action: Place a weapon at the launch site. In addition, suppose that is is also reasonable to assume that the weapon damage range $r_c \leq r_o$. The third motion strategy is based on these assumptions and the criterion: Make your position at the time of weapon placement as uncertain as possible beyond the range r_o. It is defined as follows: After missile launch, choose a course, speed and depth using the first evasive motion strategy. Maintain the course and depth throughout the evasive motion and the speed u_M for a time $t_o = r_o/u_M$ after the launch. At that time, set a speed u so that each speed between 0 and u_M is equally likely. Then vary the speed with time so that at any time $\tau > t_o$ after the launch

$$(3) \quad u(\tau) = u\, u_M\tau/\{(u/u_M)[(u_M\tau)^2 - r_o^2] + r_o^2\}^{\frac{1}{2}} \quad .$$

With this evasive motion strategy, at the time of weapon detonation, a launching submarine's position will be uniformly distributed between the two concentric circles of radius r_o and $u_M t$ that are centered on the launch site. An argument for Equation 3 is in Appendix 2.

The third weapon placement strategy is based on the criterion: Make p as large as possible given a launching submarine uses the third motion strategy. It is defined as follows: Place n weapons at points on or between concentric circles of radius $2\,r_o$ and $u_M t - r_o$ that are centered on the launch site subject to the constraint that weapon damage circles cannot overlap. For the third evasive motion strategy, at the time of weapon detonation the launching submarine is located at a random point which is uniformly distributed on a localization disc whose boundaries are concentric circles of radius r_o and $u_M t$ that are centered on the launch site. Since the area of the localization disc that is covered by weapon damage discs is $n\pi r_c$, the probability p that a launching submarine will be on or within a weapon damage circle is $n\,r_c^2/\,[(u_M t)^2 - r_o^2]$. And, since the maximum area of the localization disc that can be covered by n weapons is $n\pi r_c^2$, p is a maximum and is determined by the following equation:

$$p = n\,r_c^2/[(u_M t)^2 - r_o^2] \tag{4}$$

given the weapon placement strategy can be implemented. A necessary condition for this is that $n\,r_c^2 \leq [(u_M t)^2 - r_o^2]$.

VI. Conclusions

To the degree that the model on which the analysis in this report is based is relevant, the analysis suggests the following: If the time between launch detection and weapon detonation is expected to be short, the first evasive motion strategy should be considered. Otherwise, the second and third evasive motion strategies should be considered. The third evasive motion strategy satisfies the desire for flight from the launch point. In particular, if a submarine used the strategy and an opponent placed a weapon at the launch point and then followed the second weapon placement strategy, then p would be given by the following equation:

$$p = (n-1)\, r_c^{\,2}/[(u_M t)^2 - r_c^{\,2}] \tag{5}$$

if $r_o = r_c$. However, if $r_o \neq r_c$ or the opponent used the second weapon placement strategy without modification, the analysis is more complicated. Because of this, Equation 5 is provided only in order to supply some basis for evaluating the third evasive motion strategy under conditions where it is assumed that an opponent does not know a submarine's evasive motion strategy.

Plots of p as a function of t are shown in Figure 2 based on Equation 1 and Equation 2, in Figure 3 based on Equation 2 and Equation 4 and in Figure 4 based on Equation 2 and Equation 5.

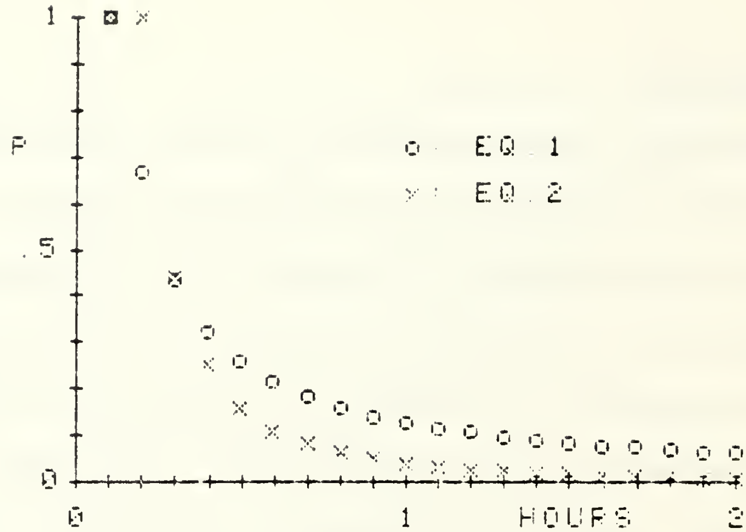

Figure 1. Plots of p as a function of t based on Equations 1 and 2 for u_M = 20 knots, r_c = 2 nautical miles and M = 4 weapons.

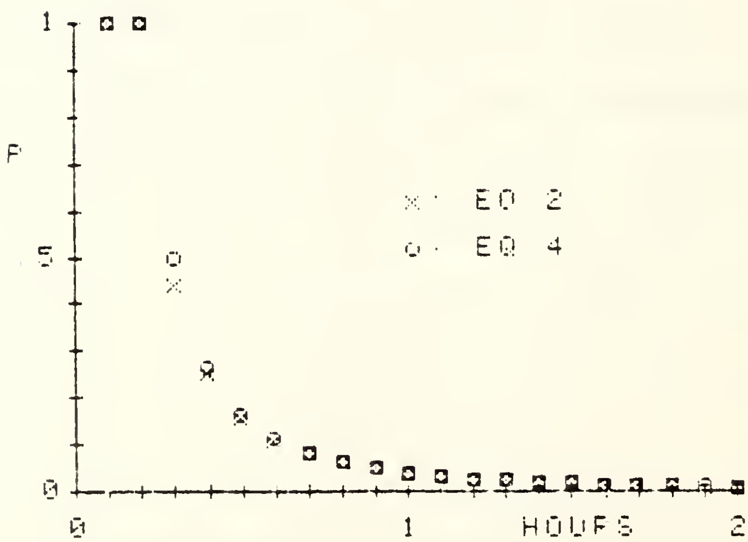

Figure 2. Plots of p as a function of t based on Equations 2 and 4 for u_M = 20 knots, r_o and r_c = 2 nautical miles and n = 4 weapons.

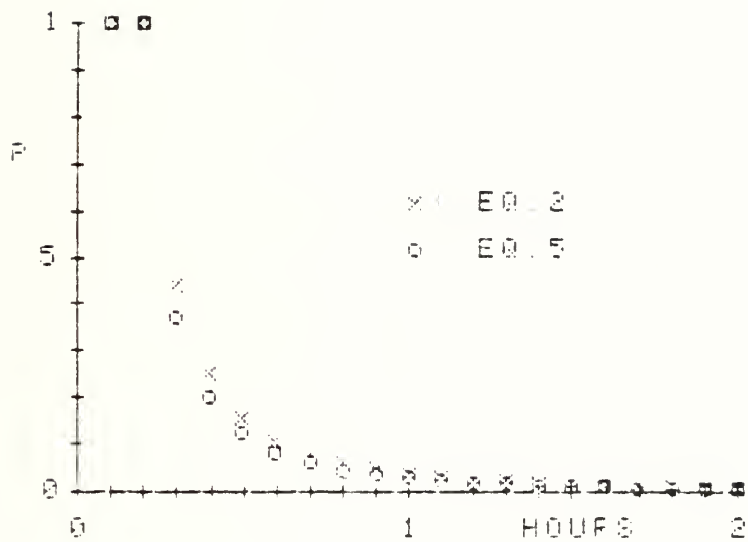

Figure 3. Plots of p as a function of t based on Equations 2 and 5 for u_M = 20 knots, r_c = 2 nautical miles, and n = 4 weapons.

The values of the parameters for the figures were chosen to provide a numerical perspective to the equations. Otherwise, they have no special significance.

Clearly, the choice of an evasive motion strategy should be ased on a more general model than that considered here. For example, the model should certainly account for geographic constraints. One way of extending the analysis would be to introduce uncertainty with respect to the damage radius r_c, an opponent's knowledge of the precise launch point and an opponent's ability to precisely determine a weapon's detonation point.

Appendix 1. The Second Evasive Motion Speed Process

With the second evasive motion model, for an opponent, at any time t after a missile launch, the launching submarine's position is to be uniformly distributed over a disc that is bounded by a circle of radius $u_M t$ centered on the launch point. To determine a stochastic process that will achieve this, take the launch point as the origin of a rectangular coordinate system. Then the submarine's coordinates at any time t after the launch will be random variables X(t) and Y(t) whose distri-bultions will be determined by a two dimensional stochastic process. In order for the submarine's position to be uniformly distributed over the disc, the joint density function of X(t) and Y(t) must be given by:

$$f_{X(t),Y(t)}(x,y;t) = 1/\pi u_M^2 t^2$$

where $X^2 + Y^2 \leq u_M^2 t^2$. With R(t) the submarine's range from the origin and $\Phi(t)$ the submarine's bearing from the origin relative to the positive y-axis, $X(t) = R(t) \sin \Phi(t)$ and $Y(t) = R(t) \cos \Phi(t)$ where R(t) and $\Phi(t)$ are random variables that are determined by a related two dimensional stochastic process. Based on this, the joint distribution of R(t) and $\Phi(t)$ is given by:

$$f_{R(t),\Phi(t)}(r,\phi;t) = r/\pi u_M^2 t^2$$

where $0 \leq r \leq u_M t$ and $0 \leq \phi < 2\pi$, since r is the jacobian of the transformation. Since the range of ϕ and r are independent, by considering their marginal distributions, it can be seen that $\Phi(t)$ and $R(t)$ are independent. In particular, $\Phi(t)$ has a uniform distributed with density function $f_{\Phi(t)}(\phi) = 1/2\pi$ where $0 \leq \phi < 2\pi$ and $R(t)$ has a triangular distribution with density function $f_{R(t)}(r;t) = 2r/u_M^2 t^2$ where $0 \leq r \leq u_M t$. The conditions on the distributions of $R(t)$ and $\Phi(t)$ can be satisfied as follows: At launch choose a speed U from a triangular distribution with density function $f_U(u) = 2u/u_M^2$ and a course θ relative to the positive y-axis from a uniform distribution between 0 and 2π. Maintain the chosen course and speed throughout the evasion. Then, since $\Phi(t) = \Theta$, the requirement on the distribution of $\Phi(t)$ is satisfied. And, since the jacobian of the transformation from the distribution of U to $R(t)$ is $1/t$, the requirement on the distribution of $R(t)$ is satisfied. Consequently, $X(t)$ and $Y(t)$ have the required distribution for any value of $t > 0$.

Appendix 2. The Third Evasive Motion Speed Process

Based on the results in Appendix 1, a submarine's position is uniformly distributed at $\tau > t_o$ between the concentric circles of radius r_o and $u_M\tau$ that are centered on the launch site if at τ the following conditions are satisfied: Its range and bearing from the launch site are independent; its bearing from the launch site is the value of a random variable uniformly distributed between 0° and 360°; and its range from the launch site is the value of a random variable $R(\tau)$ that has a cumulative distribution function given by:

$$F_{R(\tau)}(r,\tau) = [r^2 - r_o^2]/[(u_M\tau)^2 - r_o^2]$$

where $r_o < r \leq u_M$ is a value of $R(\tau)$. Now define a random variable by $P = F_{R(\tau)}[R(\tau);\tau]$. Since $F_{R(\tau)}(r;\tau)$ is a cumulative distribution function, P has a uniform distribution between 0 and 1. Next suppose that for all times $t > t_o$ the submarine's position is uniformly distributed between the concentric circles of radius r_o and u_Mt that are centered on the origin and that the $R(t)$ are correlated so that the submarine's range r varies as a continuous function of time and such that $F_{R(t)}(r;t) = p$ where p is constant and a value of P. This requires that the following equation be satisfied:

$$[(u_Mt)^2 - r_o^2]p = r^2(t) - r_o^2 \quad .$$

Differentiating Equation 7 with respect to t gives:

$$u(t) = p\, u_M^2 t/\{p[(u_Mt)^2 - r_o^2] - r_o^2\}^{\frac{1}{2}}$$

where $u(t) = dr/dt$ is the submarine's speed at t. Since $u_M t_o = r_o$ for continuity at t_o, let $u = p\, u_M$ where $u = u(t_o)$. Since p is the value of a random variable P, Equation 7 defines a random variable U(t) and $u = p\, u_M$ defines a random variable U which has a uniform distribution between 0 and u_M. Replacing p in Equation 7 by u/u_M gives Equation 3 subject to the conditions under which it is defined.

As t becomes large, the values of U(t) approach $[u\, u_M]^{\frac{1}{2}} = p^{\frac{1}{2}}u_M$ the geogetric mean of the starting speed and the maximum speed. As r_o approaches 0, the values of U(t) approach the geometric mean $p^{\frac{1}{2}}u_M$ for all $t > 0$. For $r_o = 0$ and all $t > 0$, the values of U(t) equal $p^{\frac{1}{2}}U_M$ and its cumulative distribution function is given by $F_{U(t)}(u) = u^2/u_M{}^2$ where $0 \leq u \leq u_M$. This is the cumulative distribution function of the speed for the second evasive motion strategy. Note, with respect to the continuity condition used above, with $r_o = 0$ there is a discontinuous change from a uniform distribution to a triangular distribution. However, for $r_o = 0$, the initial speed distribution could just as well have been a triangular distribution.

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